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Excitation of atomic hydrogen in the Vainshtein approximation

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The post form of the Vainshtein approximation is used to calculate cross sections for the processes $H^+ + H(1s) \rightarrow H^+ + H(2s, 2p)$. It is found that the results agree with those of McCarroll & Salin (1966) who used the prior form of the Vainshtein approximation.

1. Introduction

The Born approximation gives good results at high energies of impact but its applications are limited at low and intermediate energies. The success of close coupling approximation has also been much less marked in determining inelastic cross sections. Vainshtein *et al* (1964) pointed out that the electron-electron repulsion plays a dominant role in the excitation process by electrons and they suggested to take account of this interaction explicitly. The Vainshtein approximation has been used for a large number of electron induced transitions in hydrogen and the alkali metals (Vainshtein *et al* 1964, 1965, Presnyakev 1965). McCarroll & Salin (1966) have carried out calculations of cross sections for the processes $H^+ + H\ (1s) \rightarrow H^+ + H(2s, 2p)$ using the prior form of the Vainshtein approximation. Here attempt has been made to carry out calculations of cross sections using the post form of the Vainshtein approximation for which the analysis is given by Coleman (1969).

2. Theory

The cross section for direct excitation is

$$Q = \frac{1}{2\pi^2 v^2} \int_{q_{\min}}^{q_{\max}} |\, T_{if} \,|^2 \, q \, dq \, (\pi a_0^2) \qquad \ldots(1)$$

where $\boldsymbol{q} = \boldsymbol{k}_i - \boldsymbol{k}_f$ is the momentum transfer vector and T_{if} is the transition matrix element $<\psi_f \mid V_f \mid \psi_i^+>$. The limits of integration are $q_{\min} = k_i - k_f$ and $q_{max} = \boldsymbol{k}_i + \boldsymbol{k}_f$ where $\boldsymbol{k}_f$ is determined by the equation $\boldsymbol{k}_i^2 - \boldsymbol{k}_f^2 = \mu\, \triangle E$ in which $\mu = \dfrac{M_1(M_2+1)}{M_1+M_2+1}$ is the reduced mass of the system (M_1 = mass of proton and M_2=mass of nucleus of hydrogen atom) and $\triangle E$ the difference, in rydbergs, between the binding energies of the initial and final bound states. Vainshtein *et al* (1964) in their work on electron impact used a packing approximation and with the intention of reducing the

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error incurred in the region $s = 0$ by replacing $\phi(s)$ by $\phi(bq)$, replaced q by $-q$. For proton impact the same approximations give (Coleman 1969)

$$T_{if}^{(2)} = -Z_1 N_1(\nu_0)\, \phi(bq)\, I(\nu_0, b\mu, 0, av, q)$$

where Z_1 is the charge of the projectile,

$$|N_1(\nu_0)| = |\Gamma(1+i\nu_0)|^2 = \frac{\pi\nu_0}{\sinh \pi\nu_0}$$

($\nu_0 = Z_1\, v$, v=velocity of the projectile)

$$\phi(s) = \int \exp(is.r)\phi_f^*(r)\phi_i(r)dr$$

$$a = \frac{M_1}{M_1+1}, \; b = \frac{M_2}{M_2+1}$$

$$I(\nu, \alpha, 0, k, q) = \frac{4\pi}{T}\left(\frac{T-2\alpha\delta}{T-2\delta}\right)^{i\nu} {}_2F_1\left[-i\nu, -i\nu, 1, z\right]$$

where $T = q^2$ and $\delta = -\, q.k$.

In this case

$$z = \frac{a^2 b(q^2+\mu\triangle E)^2}{\mu\left\{q^2\left(1+\frac{a}{\mu}\right)+a\triangle E\right\}\left\{q^2(1+ab)+ab\mu\triangle E\right\}} < 1$$

and

$$T_{if}^{(2)} = -\frac{4\pi}{q^2} Z_1 N_1(\nu_0)\phi(bq) X^{i\nu_0} {}_2F_1\left[-i\nu_0 i\nu_0, 1, z,\right]$$

where

$$X = \frac{q^2(1+ab)+ab\mu\triangle E}{q^2\left(1+\frac{a}{\mu}\right)+a\triangle E} \qquad \ldots(2)$$

For proton impact $a \simeq 1 \simeq b$, $\mu >> 1$ and $z \simeq \dfrac{\triangle E}{q^2+\triangle E}$, $X \simeq \mu z$.

For excitation $\triangle E > 0$ and X as defined by (2) is positive for all relevant values of q. The cross section obtained by using $T_{if}^{(2)}$ as an approximation to T_{if} in (1) is

$$Q^{(2)} = \frac{8Z_1^2}{v^2}\int_{q_{min}}^{q_{max}} \frac{dq}{q^3}\, |\phi(bq)|^2\, |h(z,\nu_0)|^2$$

where

$$h(z,\nu_0) = \frac{\pi\nu_0}{\sinh \pi\nu_0}\, {}_2F_1\left[-i\nu_0, i\nu_0, 1, z\right].$$

After some simplifications the cross sections for $2s$ and $2p$ excitations are

$$Q^{(2)}(s) = \frac{2^{22}\pi^2}{v^4(e^{\pi/v}-e^{-\pi/v})^2}\int_{q_{\min}}^{q_{\max}} \frac{q\,dq}{(4q^2+9)^6}\left|\,{}_2F_1\left[\frac{-i}{v},\frac{i}{v},1,\frac{\triangle E}{q^2+\triangle E}\right]\right|^2$$

and

$$Q^{(2)}(p) = \frac{2^{20}\times 9\pi^2}{v^4(e^{\pi/v}-e^{-\pi/v})^2}\int_{q_{\min}}^{q_{\max}} \frac{dq}{q(4q^2+9)^6}\left|\,{}_2F_1\left[\frac{-i}{v},\frac{i}{v},1,\frac{\triangle E}{q^2+\triangle E}\right]\right|^2$$

where $q_{\min}=k_i\left[1-\left(1-\frac{\mu\triangle E}{k_i^2}\right)^{\frac{1}{2}}\right]$

3. Results

The integration over q has been carried out numerically. In the present work z always lies in the range $0 < z < 1$. The series development of the hypergeometric function is given by

$$ {}_2F_1\left[a,b,c,x\right] = 1+\frac{ab}{c}x+\frac{a(a+1)b(b+1)}{c(c+1)1\times 2}x^2+\ldots \tag{3}$$

It has been found that convergence of this series is good for all relevant values of q and velocities of projectile used. In the present calculations the series expansion given by (3) is used and all the terms of the series are real.

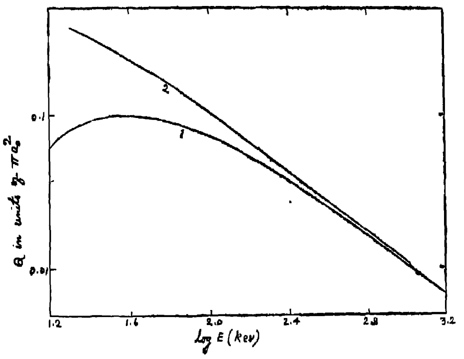

Figure 1. Cross sections for $H^+ + H(1s) \rightarrow H^+ + H(2s)$
Curve 1. Vainshtein approximation
Curve 2. Born approximation

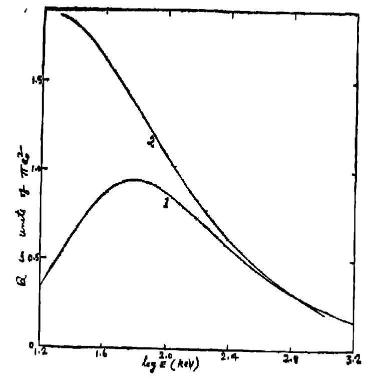

Figure 2. Cross sections for $H^{+}+H(1s)\rightarrow H^{+}+H(2p)$
Curve 1. Vainshtein approximation
Curve 2. Born approximation

TABLE 1 CROSS SECTIONS FOR $H^{+}+H(1s)\rightarrow H^{+}+H(2s)$ EXCITATION IN UNITS OF πa_0^2

E(kev)	Cross Section	E(kev)	Cross section
16.0	6.26, - 2	156	5.36, - 2
25.0	9.04, - 2	225	4.05, - 2
36.0	9.92, - 2	400	2.48, - 2
42.2	9.90, - 2	624	1.65, - 2
56.2	9.36, - 2	899	1.17, - 2
63.9	8.96, - 2	1224	8.73, - 3
80.9	8.08, - 2	1599	6.74, - 3
99.9	7.20, - 2		

TABLE 2 CROSS SECTIONS FOR $H^{+}+H(1s)\rightarrow H^{+}+H(2p)$ EXCITATION IN UNITS OF πa_0^2

E(kev)	Cross section	E(kev)	Cross section
16.0	3.44, - 1	90.2	9.02, - 1
25.0	6.38, - 1	99.9	8.79, - 1
30.2	7.50, - 1	156	7.44, - 1
36.0	8.33, - 1	225	6.19, - 1
42.2	8.90, - 1	400	4.37, - 1
49.0	9.24, - 1	624	3.22, - 1
56.2	9.40, - 1	899	2.48, - 1
63.9	9.43, - 1	1224	1.96, - 1
72.2	9.36, - 1	1599	1.60, - 1
80.9	9.22, - 1		

4. Discussion

It is clear from the figures that at high energies the Vainshtein approximation is in close agreement with the Born approximation. At lower energies the Vainshtein approximation cross sections are smaller than the corresponding Born approximation cross sections. The maxima are displaced considerably to higher energies as compared with the Born approximation results. It is found that the present results agree with those of McCarroll & Salin (1966) who used the prior form of the Vainshtein approximation. It is also found that in case of $1s \rightarrow 2p$ transition the Vainshtein approximation cross sections are slightly larger than the Born appoximation cross sections at high energies. It is possible that the Vainshtein approximation results can be improved by introducing an effective charge (Vainshtein *et al* 1964, Crothers & McCarroll 1965).

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